A NEW INSTRUMENT FOR WEBER'S LAW; WITH INDICATIONS OF A LAW OF SENSE MEMORY.

By James H. Leuba.

The three methods generally used in the study of the relation between physical stimuli and the sensations they produce—the Method of Least Observable Difference; the Method of Average Error; the Method of Right and Wrong Cases—are only different ways of reaching the same end, viz., the determination of the least observable difference in sensations.

Delbœuf and Wundt have attempted to verify Weber's Law by following another road. Instead of seeking to measure the least preceptible increment of sensation at different points of the scale of intensities, they seek to determine a middle sensation between two others, varied at will. This Method of Equal Intervals *(Methode der mittleren Abstufungen)* admits of a modification of some importance; it consists in the classification between two limits of a series of sensations into groups, in such a way that each group will appear to be at an equal interval from the one immediately preceding, and from the one immediately following it.

The comparison of the magnitudes of the stars, as determined by the eye, and recorded in the various catalogues, with the photometric measurements of their light intensities, is an application of this method, if, indeed, we can assume that the sole desire of the star-catalogue makers was to classify the stars into magnitudes, each appearing equally distant from the next.[1]

This method is evidently capable of wide and useful application, equally well in the measurement of intensities as in that of extensity. Prof. James, contrasting the Method of Equal Intervals, of which the Classification Method is a species, with the three usual methods, says: "At first sight there seems to be no direct logical connection between this method and the preceding ones. By them we compare equally

[1]The last comparison of this kind was made by Prof. Jastrow. See *Amer. Journal of Psychology*, Vol. I. p. 112.

perceptible increments of stimulus in different regions of the latter's scale, but by the fourth method we compare increments, which strike us as equally big. But what we can but just notice as an increment, need not appear always of the same bigness after it is noticed. On the contrary, it will appear much bigger when we are dealing with stimuli that are already large."

If the relation of the sensations to the stimuli producing them is found by the Method of Equal Interval (or the Classification Method) to be the same as when established by one of the Methods of the Least Observable Difference, the just perceptible increment in sensations will have been proved to remain equally big in the observer's consciousness. This question is in itself of sufficient interest to warrant experimentation with the Interval Methods.

Prof. Jastrow is, so far as we know, the only person who has made use of the Classification Method. He applied it in the study of star-magnitudes, just mentioned, and in experiments on the spatial relations of vision, on the tactile-motor sensations, on the time-sense, and on the motor-sense.[1]

In order to adapt this psycho-physic method to demonstrational purposes, Dr. Sanford last year devised an instrument (Fig. 1) for the production and the measurement of artificial stars of different magnitudes, using the principle of the episkotister for regulating the amount of light passing through a minute hole in a metal plate.

The present paper records the experiments made with this device, according to the Classification Method, to test the possible usefulness of the instrument.

Two lots of results were obtained, one with the apparatus just mentioned (Fig. 1), the other with a modification of it, suggested by the writer (Fig. 2).

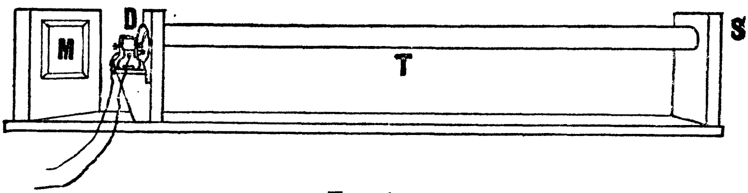

FIG. 1.

The most important part of the first apparatus is the episkotister, two equal discs (D), four inches in diameter, set on the axis of a small electric motor. Each of them has two openings of 90°, in opposite quadrants. The discs can be so placed in relation to each other that their openings will coincide on

[1] See *American Journal of Psychology*, III. and IV.

any angle between 0° and 90°. To permit of exact adjustments, one of them is graduated along one of its openings. A light thrown on the mirror M, is reflected towards the discs, and as they rotate, a quantity of light proportional to the opening passes through to a very small hole, bored in metal, at the extremity of the tube T, and covered over on both sides by pieces of glass to keep it free from dust. The subject, seated at the other end of the tube and looking in at S, perceives the light in the form of a star. The position of the observer can be fixed by reducing the opening of the tube so that he will have to sit with one eye close to the small space left free in order to see the light. The tube T is about six feet long. By means of a switch, the motor can be readily stopped for the readjustment of the discs.

The two extreme stars used as standards were made by an opening of 180° and 10° respectively. The latter is barely perceptible; the former is about the size of the smallest stars of the first magnitude. At the beginning of a new sitting, the subject was shown the two standards, and also during its course, whenever he desired to see them again, though this was rarely asked for. In each series of experiments the subject was shown the forty stars made by the forty openings, between 10° and 180°, which are given in degrees in the degree columns of Table I. The series was so chosen that about the same number of stars would fall into each of the five classes if the psycho-physic law were followed. They were shown in an irregular order, and the subject was requested to group them in five classes, or magnitudes, endeavoring to make the differences between the classes equal. His answers were recorded in tables similar to Table I., opposite the figures in the first column representing the measure of the intensities of the lights. The numerals stand for the class to which the star was referred.

TABLE I.

Degrees.	1st Series.	2d Series.	3d Series.	4th Series.	5th Series.	Degrees.	1st Series.	2d Series.	3d Series.	4th Series.	5th Series.	Degrees.	1st Series.	2d Series.	3d Series.	4th Series.	5th Series.	Degrees.	1st Series.	2d Series.	3d Series.	4th Series.	5th Series.	Degrees.	1st Series.	2d Series.	3d Series.	4th Series.	5th Series.
180	1	1	1	1	1	112	2	2	2	1	2	64	2	3	2	3	2	32	3	3	4	4	3	16	3	4	4	5	5
170	1	1	2	1	1	106	2	2	2	1	2	60	2	3	3	2	2	30	4	3	4	4	3	16	4	3	5	5	3
160	1	1	1	1	1	100	3	1	1	1	1	56	3	2	2	3	2	28	4	4	3	3	5	14	4	4	4	4	3
152	2	1	1	1	1	94	1	2	2	1	1	52	2	1	2	3	2	26	4	3	4	4	4	14	5	4	5	5	5
144	1	2	1	2	1	88	3	2	2	3	3	48	4	3	3	3	2	24	3	3	3	4	5	12	5	5	5	5	5
136	1	1	1	1	1	82	1	2	2	1	1	44	2	2	2	2	4	22	4	4	5	5	2	12	4	5	5	5	5
128	2	1	1	1	1	76	2	3	2	3	1	40	3	2	3	5	3	20	3	4	3	4	4	10	5	5	5	5	5
120	2	1	2	2	1	70	3	3	2	1	2	36	3	2	3	2	3	18	4	4	3	4	4	10	5	5	5	5	5

Table II. summarizes Table I., giving the average of each class in each of the five series, and, at the bottom, the average ratios between the classes. The subject has attempted to place his sensations in an arithmetical series of five terms; we have here the objective measurement of the stimuli which produced these sensations; if Weber's Law were exactly followed, the ratios should be equal.

TABLE II.—CLASSES.

SERIES.	I.	II.	III.	IV.	V.
1st.	138. (7)	91.4 (10)	48.2 (10)	23.7 (9)	11.5 (4)
2d.	133.1 (9)	80.2 (10)	44.6 (10)	18.8 (7)	11. (4)
3d.	142.8 (7)	87.2 (13)	34.2 (8)	26. (4)	13.7 (8)
4th.	125.6 (12)	80.8 (5)	58.8 (7)	23.4 (7)	16.9 (9)
5th.	128.5 (12)	65.5 (9)	36.5 (7)	27.0 (4)	15.7 (8)

	I.—II.	II.—III.	III.—IV.	IV.—V.
RATIOS.	1.66	1.88	1.91	1.75

The class averages given in this table are arithmetical averages. It would have been more correct to follow the indications of the psycho-physic law, and to take the geometrical mean, but as the results obtained by these two methods are very nearly the same, we chose the former to avoid the great labor of calculating the geometrical averages. The numbers in parentheses at the right of the class-averages indicate the number of stars which enter the classes.

Table III. gives the average ratios of all the judgments passed by the nine persons who served as subjects in the first lot of experiments. It involves the classification of 2,120 stars (53 series). The last row of figures represents the final average-ratios of all the results, weighed by the number of series. We shall discuss its significance later.

TABLE III.

CLASSES.	I.—II.	II.—III.	III.—IV.	IV.—V.
Be.—4 s.	1.70	1.94	1.89	1.77
Bo.—12 s.	1.58	1.80	2.18	1.75
Br.—3 s.	1.76	1.96	1.92	1.66
D.—3 s.	1.54	1.97	2.28	1.67
F.—6 s.	1.45	2.21	1.81	1.82
K.—6 s.	1.66	1.90	1.88	1.79
L.—5 s.	1.66	1.88	1.91	1.75
R.—7 s.	1.76	1.87	1.86	1.72
S.—7 s.	1.82	1.99	1.87	1.58
Averages (Weighed).	1.66	1.93	1.96	1.73

At this point the experiments were interrupted by the summer vacation of last year. At the beginning of 1893 Dr. Sanford asked me to complete them.

The falling off of the ratio at both ends of the scale of intensities, for which we could not account (see total average-ratios in Table III.), suggested changing the position of the standards from the extremes (10° and 180°) to a point near the middle of the first and the last classes.

Six series of experiments were taken with the standards at 20° and 160°, and the standards were shown at the beginning and regularly after every five judgments.

TABLE IV.

CLASSES.	I.—II.	II.—III.	III.—VI.	IV.—V.
Leu.—3 s.	1.41	1.76	2.10	1.99
S.—3 s.	1.49	1.45	2.11	1.93
Averages.	1.45	1.61	2.12	1.96

The results (Table IV.) are not based on a sufficient number of experiments to afford a sure basis of comparison with Table III., and in addition to this source of uncertainty, the acquaintance of the two subjects with the scheme of classification had a biasing influence, for in one case, for instance, the subject observed that he was attempting to avoid a mistake which he knew he had made in a previous series, thus judging no more solely from his sensations. The knowledge of the position of the standards in the scale of intensities had also a disturbing effect. The order in which the stars were presented was observed to influence the results. If many stars of about the same intensity were shown successively, the subject would lose sight of the true extent of the scale, and consequently make false judgments. This source of error could not be very great when the standards were shown frequently. The results of these six series do not differ greatly, however, from those of Table III.

The absence of the standards appeared to me the source of a considerable error. During the intervals between their appearance, the subject had in mind a representation of them, unavoidably erroneous in some degree. Even when they were shown regularly at relatively short intervals, as in the last experiments, it often happened that the observer would say, when looking at one of the standards: "I see that I was wrong in my last judgment," thus plainly indicating that he had forgotten the magnitude of the standards. This source of error would naturally bear specially on the last class.

To obviate this defect and to make the conditions of the experiment more nearly those of real star classifications, a new apparatus was made (Fig. 2), in which the standards would

constantly be in sight. The star-making device was also modified in order to allow of the production of a greater number of stars between the limits. A disc (D) seventeen inches

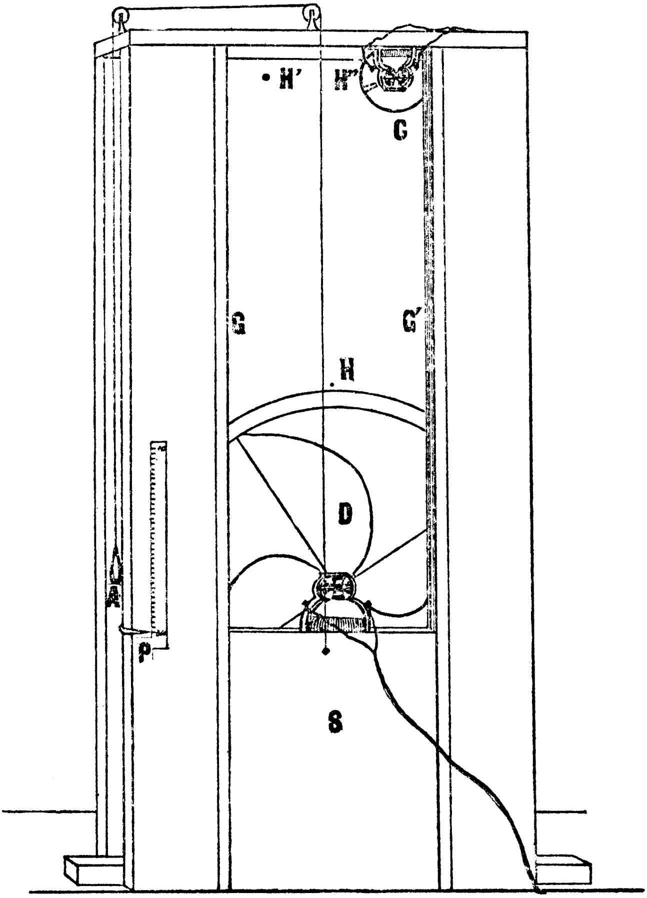

FIG. 2.

in diameter was so cut that as the centre is approached the amount of filled space increases. (See Fig. 3). It is rotated by a small electric motor, fixed on a slide (S). The operator, by means of the handle (A), moves the slide, and with it the disc (D) between the guides (G G′). The position of the disc with reference to the pin-hole (H), through which the

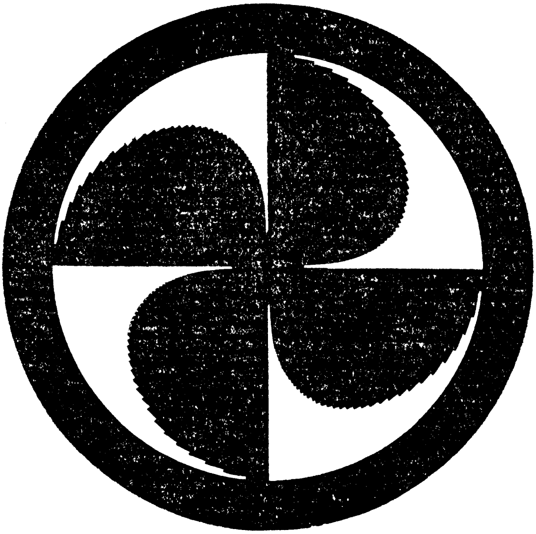

FIG. 3.

light passes, is indicated by the pointer (P), which moves with the slide.[1] The disc can thus be held in fifty different positions, corresponding to the fifty openings expressed in degrees in the figures in the degree columns of Table V. The drawing gives a back view of the apparatus. The subject sat in front, at a distance of about two metres, in a position fixed by a head-rest.

Two other pin-holes (H′) and (H″), the latter hidden by the rotating disc (C), equal in size with (H), give the two standards: (H′), the superior standard, by the free access of the light, and (H″), the inferior, by means of a slit in the disc (C). The light is so placed that it falls equally on the three holes, but before reaching them it passes through a piece of oiled paper, stretched at a little distance from them. Kerosene

[1] It is essential to accuracy that the disc should revolve very close to the hole, especially if the light is not in the same horizontal plane with it. When the disc is at some distance from the hole, and the light comes from above, it may reach the hole through a division of the disc above the one recorded as the measure of the magnitude of the star. A slight error, amounting to a notch of our disc, thus found its way into this second series of experiments. As their purpose, however, was chiefly to show the adaptability of the method to general laboratory use, and as the constant error introduced does not interfere at all with the very general inferences drawn from them, it has seemed best to let them pass.

lamps were used as the steadiest light available. The experiments were conducted in a dark room, the light of the lamps being projected in the direction of the apparatus only.

The instructions to the subject were different from those in the previous experiments. Again he was to classify into five groups at equal intervals from each other, but the standards were now used as limits only, and were not to enter into the classes. When a star appeared to be equal to one of the standards, the subject was to designate it as the Superior or Inferior Limit. He had thus before him a sharply bounded scale of intensities.

This apparatus could also be used for the Method of Equal Intervals. The magnitude of star (H″) can be altered at will by changing the episkotister rotating in front of the hole, as explained in the description of Apparatus I. The subject's task then becomes to find a star at an equal distance in intensity from (H′) and (H″).

Before going further, let me insert here a few observations of some importance in conducting the experiments:

Contrast. In order to avoid contrast effects, the lights used as standards should not be too near the star to be classified. In our apparatus they were at a distance of about fifty cm.

After-Images. The after-images of the standards have no doubt, in some degree, a disturbing influence. Fixating the lights longer than necessary should be avoided. A mere glance, repeated three or four times at short intervals when necessary, was found to yield the best results. In this manner the judgment was often made in memory, when the stars to be classified, as well as the standard, were no more before the eyes. The eyes should be closed between the judgments.

Fatigue varied greatly with the person and with the circumstances. But the irregularities resulting from fatigue were always great enough to render worthless observations taken without regard to it. If comparisons were made in rapid succession for a few minutes, the subject lost confidence in his classifications, and his judgments became clearly erroneous. He would place the same star, seen at a very short interval, in widely different parts of the scale of intensities. For instance, the same star was located now in the second class, and now in the fourth.[1]

A rest of two or three minutes reëstablishes certainty of judgment. When in doubt as to the proper place of a star, the longer one gazes at it continuously, the more confused one becomes. To avoid the effects of fatigue, the stars should

[1] After-images may play an important rôle in this confusion.

be shown at intervals, varying with the circumstances, and the series of fifty should generally be cut in two, a rest of some half hour being allowed between the two halves.

The results of the first series at least should be rejected. The subject does not realize at the beginning the extent of the scale which he is to divide into five, and, as he advances in the classification of his first series, he very generally perceives that he has made too large or too small a place for one class to the detriment of the others.

The ratios of subject A., in Table VII., decrease with the magnitude, while those of all the other subjects, whose observations are recorded in Table VII., increase with the decrease of the magnitude. The subject stated that knowing the decrease in the relative discriminative power near the limit of visibility, he modified his judgments accordingly, in order to make the five classes differ from each other by the same quantity. This interference with the sensations accounts sufficiently for the inversion of the common order. The ratios without the judgments of this subject are given at the foot of Table VII.

Extension and Color. The stars appeared to be of different extensions, although the holes were of the same size. The brightest star was judged to cover a surface about ten times larger than the one occupied by the faintest. Some of the subjects noticed that their classification was somewhat modified by this extension element. Irradiation, and, perhaps, an illusion of greater surface, produced by a greater intensity of light, may be the cause of this phenomenon.[1] With the kerosene lamps the color of the stars took a yellowish tint as the magnitude increased. This change of color, although very slight, may have influenced the classification.

Let us now take up the group of results obtained with Apparatus II., and compare them with the first group.

Table V. shows the classification of four series, as made by subject Bo.; Table VI. gives the averages of each class in each of these four series, and, at the foot, the ratios. Finally, in Table VII. will be found the class-ratios, based on 1,100 judgments (22 series), made by five persons. At the foot of Table VII. are the general averages weighted, as before.[2]

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[1] A certain complication of conditions is thus introduced, for, the relation of intensity of stimulus to sensation and of the quality of stimulus to the same, are not identical, but in this the classification of artificial stars stands on the same footing as that of real stars.

[2] In two series of subject Be. (Table VII.), who very soon showed the signs of fatigue, the judgments were taken only at the critical points between magnitudes.

TABLE V.

Subject Bo.

Degrees.	1st Series.	2d Series.	3d Series.	4th Series.	Degrees.	1st Series.	2d Series.	3d Series.	4th Series.	Degrees.	1st Series.	2d Series.	3d Series.	4th Series.	Degrees.	1st Series.	2d Series.	3d Series.	4th Series.	Degrees.	1st Series.	2d Series.	3d Series.	4th Series.	Degrees.	1st Series.	2d Series.	3d Series.	4th Series.	Degrees.	1st Series.	2d Series.	3d Series.	4th Series.
360	2	2	1	2	216	2	3	3	3	129	3	3	3	4	77	4	4	4	4	46	5	5	5	5	28	5	6	6	6	16.5	6	6	6	6
334	2	2	2	2	200	2	3	2	3	120	3	4	3	4	72	5	4	4	4	43	5	5	5	5	26	6	6	6	6	15.5	6	6	6	6
311	1	2	2	2	186	3	3	3	3	112	4	4	3	4	67	4	5	4	4	40	5	5	5	5	24	6	6	6	6	14.5	6	6	6	6
289	2	2	2	2	173	3	3	3	3	104	3	4	4	4	62	5	5	4	5	37.5	5	5	5	5	22	6	6	6	6	13.5	6	6	6	7
269	2	2	2	2	161	3	3	3	3	96	4	4	4	4	58	4	5	5	5	35	6	6	5	6	20.5	6	6	6	6	12.5	7	6	6	6
250	2	2	2	2	150	3	4	3	3	89	4	4	4	4	54	5	5	5	5	32.5	5	6	5	5	19	6	6	6	6	11.5	7	7	7	6
232	2	3	2	2	139	3	3	3	4	83	4	4	4	4	50	5	5	5	5	30	5	6	6	6	17.5	6	6	6	6	10.75	7	7	6	7
																														10	7	7	7	7

TABLE VI.

SERIES.	Superior Limit.	I.	II.	III.	IV.	V.	Inferior Limit.
1st.	311 (1)	268.8 (8)	145.5 (8)	83.1 (7)	45. (11)	21.3 (11)	11.2 (4)
2d.		302.2 (6)	179.5 (8)	100.4 (9)	50.9 (9)	21.5 (15)	10.7 (3)
3d.	360 (1)	269.3 (7)	154. (9)	81.3 (8)	44. (9)	19.3 (14)	11.3 (2)
4th.		292.1 (7)	181. (6)	98.9 (11)	47. (9)	20.9 (14)	11.4 (3)
Averages.		283.1	165	90.9	46.7	20.8	11.2
Ratios.			1.72	1.82	1.94	2.25	

TABLE VII.

CLASSES.	I.—II.	II.—III.	III.—IV.	IV.—V.
A.—4 s.	2.06	2.04	2.05	1.73
Be.—4 s.	1.85	1.77	2.01	1.93
Bo.—4 s.	1.72	1.82	1.94	2.25
Leu.—6 s.	1.75	1.82	1.91	2.36
T.—4 s.	1.84	1.81	1.70	2.02
Averages (Weighted).	1.83	1.85	1.92	2.09

Averages Omitting Series of A.

	1.78	1.81	1.89	2.16

The reader will notice that in the general average the ratio of the average intensity of each class, to that above it, increases as the brightness of the stars decreases, and indications of the same thing appear in some of the individual records of both groups. It is greatest between the classes made up of the faintest stars, and it is smallest between Classes I. and II., where the magnitude is greatest. Since the lower limit of our scale of intensities was a barely perceptible light, and the upper limit star much below the superior light intensity, our results agree well, as far as comparison can be made with those obtained by experimenters who used the Methods of Least Differences.

Prof. Jastrow, in the articles on star magnitude, mentioned above, reaches an opposite conclusion. He finds that "the law regulating the ratio of light between stars of one magnitude and those of the next above it, is the psychophysic law as formulated by Fechner, with the modification, however, that the ratio in question, instead of being perfectly constant, decreases slightly with the brightness of the star."[1]

Other persons (Wolff, Pierce, etc.) obtained similar results by comparing special star-catalogues with photometric measurements.

[1] The extreme ratios are 2.802 between the second and the first, and 1.876 between the seventh and the sixth magnitudes.

A little surprise at this is natural when it is remembered that all the observations not based on star-catalogues show that the relative discriminative sensibility falls when weak or over-strong lights are compared. (See the experiments of Aubert, Masson, Helmholtz, and König and Brodhun.) Moreover, this deviation from Weber's Law is general, and applies also to other sensations than to those of light. Biedermann and Löw, experimenting with weights between 10 and 500 gr., found that the sensitiveness to pressure rose with the increase of weight from ten to 400 gr. and then fell rapidly.

Inasmuch as the scale of intensity began with the seventh magnitude, that is to say, with stars perceptible only to acute sight, it was to be expected that the discriminative powers would be proportionately less in the lower magnitudes, and that, consequently, the ratios between the classes would decrease with the increase of the light intensity.

This constant and well-defined disagreement between the persons who dealt with lights produced and measured for their purpose, and those who started from the star-catalogues, indicates, perhaps, that the early astronomers were influenced in their star classification by some other consideration than the desire to make each magnitude equally different from the next. As the number of the stars is much greater in the lower magnitudes, the lower classes may have been made narrower in range for practical purposes. This is, indeed, a very plausible explanation; what the early astronomers wanted, first of all, was a convenient grouping of the stars, and, although magnitude was, no doubt, taken as the basis of the classification, it seems highly probable that the great difference in the number of stars belonging to the different magnitudes should have modified it, consciously or unconsciously, in the direction indicated by the comparisons considered. When we remember how strong was a similar tendency in some of our subjects, we are inclined to say that it could not have been otherwise.

Indications of a Law of Sense-Memory. If we now compare the average class-ratios of Table III. with those of Table VII., we shall see that the most striking difference is the fall from 1.96 to 1.73 in the ratio at the lower end of the scale, in the results of the experiments made with the first apparatus. This fall finds a ready explanation in the absence of the standards while the comparisons and the judgments were made. There seems to be a natural tendency in us to shift the sensations held in memory towards the middle of the scale of intensities. It might be conceived to operate somewhat as follows. The image of a recent sensation tends to recall, by

association, the united residual of all the past sensations of the same kind, and in so doing passes over, in some degree, to this sub-conscious resultant impression. Something of this kind has long been recognized as a law of the recollection of extreme emotional states, but it has apparently escaped notice as a law of sense-memory also. Something similar was observed by Mr. Bolton in the experiments reported in the first paper of this series (see p. 304); and the experiments of Dr. Nichols also, (AMER. JOUR. PSYCHOL., IV., 1891–92, pp. 75–79), showing, as they do, that practice upon a given rate of tapping tends to draw other rates of tapping toward the practiced rate, give a certain amount of support to the opinion that memories generally tend toward what has been most frequently experienced before.

The amount of modification of the sensations in memory, on account of this tendency, should vary with the time separating the perception of the sensations from the comparison, and with their position in the scale. Starting from a middle range of intensity with a deviation equal to zero, the tendency should increase as the extremes are approached.

Supposing this to be true, we should expect that the lower standard used with Apparatus I., being a just perceptible star, would assume in the subject's memory a magnitude greater than the real. This very plausible hypothesis received confirmation in the second lot of our experiments, in the fact that the subjects generally designated as equal to the lower limit stars superior to it in intensity, although the standard was always in sight; so that the average limit, as determined by the judgments of the subjects, was 11° 38′ instead of 10°, the measure of the real limit. If the error can amount to nearly one-seventh of the stimulus, when comparison is always possible, that is, when the time elapsing between the perception of the standard and that of the object to be compared is very short, it will not be surprising to find that it rises to about one-fifth when the comparison is made with an intensity of light experienced from a few seconds to five minutes, or even more, before the judgment is passed, as in experiments recorded in Table III.

This displacement of the standard in memory would affect chiefly the last class, but also all the others up to that unknown point or region which we called the middle of the scale of intensities.

This change occurring to the images of sensations, if further experiments prove it to be a universal one, must be taken into account in all experiments involving the successive comparison of sensations; for, whatever their nature, what the subject would have in mind at the time of the comparison

would not be the image of the standard, but another image differing from it in intensity and possibly in form.

Its reality might easily be tested, and, if it proves real, it might be measured by the Method of Right and Wrong Cases (though the experiments would present difficulties), and the construction of the curves representing the displacement in memory of the different species of sensations toward the middle of the scale of intensities, for various lengths of time, would be of considerable value. The influence of this phenomenon on the results of our experiments with Apparatus II. was to decrease the distance, and consequently the ratios, between the classes, as we passed from stronger to weaker stimuli. But as the star-limits were always in sight, the amount of error from this source was comparatively small.

Regarding the results of both groups of experiments from the standpoint of Weber's Law, it is evident that the complete uniformity of ratio required is not shown. The deviations, however, are not extremely great, and the series of magnitudes is very much more nearly a geometrical series than an arithmetical. We feel justified, therefore, in recommending some such apparatus as has been described as a means of introducing star-classification into the laboratory as a psycho-physic experiment.